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NATURAL AND FORCED CONVECTION HEAT TRANSFER FROM A PLATE FIN

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Abstract—An analysis is made of heat transfer from a plate fin, which is cooled by forced or natural convection. Approximate expressions are used to relate convective heat flux and temperature in the case of laminar and turbulent boundary layer in forced flow and in the case of natural convection in a vertical fin. A simple solution procedure is presented to solve conjugated heat transfer composed of conduction in a fin and convection from it. Meaningful heat transfer results are obtained when heat transfer of an actual fin is compared with an ideal isothermal fin. It was found that one curve is enough to present heat transfer results without parameters.

NOMENCLATURE

c,	constant for natural convection in equation (4);
C,	constant for turbulent convection in equation (1);
g,	acceleration of gravity;
k,	thermal conductivity of fin;
k_f,	thermal conductivity of fluid;
l,	transverse length of fin;
m, n,	constants in equation (1);
Pr,	Prandtl number;
q,	convective heat flux;
Re,	Reynolds number, $U_\infty x/\nu$;
s,	dummy variable in vertical direction;
t,	fin half thickness;
T,	fin temperature;
T_w,	fin base temperature;
T_∞,	free stream temperature;
U_∞,	free stream velocity;
x,	vertical coordinate;
x_*,	dimensionless coordinate for natural convection, equation (17);
X_*,	dimensionless coordinate for forced flow, equation (10);
y,	transverse coordinate;
y_*,	dimensionless coordinate, y/l;
α, γ,	constants in equation (1);
β,	volumetric expansion coefficient;
ν,	kinematic viscosity;
θ,	dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$;
ϕ', ϕ,	local and total heat flux of fin;
ϕ_i', ϕ_i,	local and total heat flux of an ideal isothermal fin;
ΔT,	temperature difference, $T-T_\infty$;
ΔT_w,	base temperature difference, T_w-T_∞.

1. INTRODUCTION

HEAT transfer analysis of extended surfaces with different shapes is well known for the case, when heat transfer coefficients are given (see e.g. Kern and Kraus [1]). Even for a prescribed varying heat transfer coefficient solution can be easily obtained numerically. In actual practice, however, convective heat transfer from a fin and conduction along it can not be solved separately, but they are coupled together. The problem considered in this paper is schematically shown in Fig. 1, which presents a plane fin of thickness $2t$ and

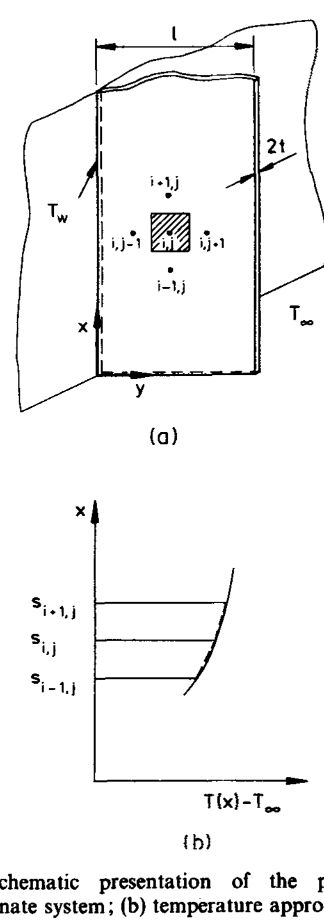

FIG. 1. Schematic presentation of the problem: (a) coordinate system; (b) temperature approximation.

transverse length l. The base surface temperature is uniform and equal to T_w and the fin is cooled by a fluid at temperature T_∞.

The analysis presented in this article deals with heat transfer of a plane fin cooled by forced or natural convection. In the case of natural convection only a vertical fin is considered. For forced convection the freestream velocity is equal to U_∞ and both laminar and turbulent boundary layers are included in the analysis.

In the problem considered in Fig. 1(a) temperature of the fin varies in the transverse direction and also it is dependent on x. Convective heat transfer in the flow direction could be solved if surface temperature is known. Unfortunately, temperature distribution is also affected by heat conduction in the fin. Thus we are dealing with a complex conjugated heat transfer problem, the solution of which can only be obtained by solving conduction in the fin together with convective heat transfer.

In this investigation an approximate treatment of natural and forced convection is used to relate heat flux and temperature. A simple procedure is presented to solve the arising complex nonlinear differential equation. It is noteworthy that dimensionless heat transfer results of forced convection and also those of natural convection on a vertical fin are free of parameters.

2. HEAT FLUX DISTRIBUTION FOR ARBITRARY SURFACE TEMPERATURE

First the expressions of heat flux distribution resulting from a known arbitrarily varying surface temperature are given. These equations are used in the forthcoming presentation when the solution procedure of conjugated heat transfer in the fin is developed.

Forced convection

In the case of forced convection it is well known that the resulting heat flux corresponding to arbitrarily varying surface temperature can easily be obtained using an integral method and superposing technique. It can be expressed quite accurately lor laminar and turbulent boundary layer if $Pr > 0.6$ as [2]

$$q(x) = C\frac{k_f}{x}Re^m\,Pr^n \int_0^x [1-(s/x)^\gamma]^{-\alpha}\,\mathrm{d}T_s(s). \quad (1)$$

The constants in equation (1) are for laminar boundary layer: $C = 0.332$, $m = \frac{1}{2}$, $n = \frac{1}{3}$, $\gamma = \frac{3}{4}$ and $\alpha = \frac{1}{3}$. The corresponding values of constants for turbulent boundary layer are: $C = 0.0296$, $m = \frac{4}{5}$, $n = \frac{3}{5}$, $\gamma = \frac{9}{10}$ and $\alpha = \frac{1}{9}$. Integration of equation (1) is easily made when surface temperature T_s is approximated by a series of straight lines as in Fig. 1(b) [2]. The integrated form of equation (1) needed later is

$$q(x) = C\frac{k_f}{x}Re^m\,Pr^n\left\{\Delta T_0 + x \times \sum_{i=1}^{n} k_i\left[G\left(\frac{s_i}{x}\right) - G\left(\frac{s_{i-1}}{x}\right)\right]\right\}. \quad (2)$$

In equation (2) ΔT_0 is the temperature difference at the leading edge of the plate, $k_i = (T_i - T_{i-1})/(s_i - s_{i-1})$ and

$$G\left(\frac{s_i}{x}\right) = \int_0^{s_i/x} [1-\xi^\gamma]^{-\alpha}\,\mathrm{d}\xi. \quad (3)$$

Natural convection

In the case of natural convection from a vertical surface such a simple method as in forced flow, in which case heat flux resulting from a prescribed surface temperature could be found, is not possible. Fortunately, Raithby and Hollands [3] have succeeded in obtaining an approximate equation relating surface temperature and heat flux. They used the analogy between a condensate film and the inner part of boundary layer in natural convection. For a vertical surface with laminar boundary layer their result can be written as

$$q(x) = ck_f\left(\frac{g\beta}{\nu^2}Pr\right)^{1/4}\Delta T^{5/3}\left[\int_0^x \Delta T^{5/3}\,\mathrm{d}x\right]^{-1/4} \quad (4)$$

which is applicable for $Pr > 0.6$. The constant c in equation (4) can be calculated very accurately using [4]

$$c = \frac{0.503}{[1+(0.492/Pr)^{9/16}]^{4/9}}. \quad (5)$$

In Raithby and Hollands [3] a more complex expression than equation (5) is used. In the case of a polynomial surface temperature x^n the accuracy of equation (4) can be compared with the exact one. In the case of a constant surface temperature equation (4) gives an exact result, but when $n > 0$ it underestimates heat flux. For instance, when heat flux density is constant ($n = \frac{1}{5}$) it gives the error of 4% with $Pr = 1$.

Equation (4) can be easily integrated if the variation of temperature distribution is composed of straight lines as in forced flow. It gives

$$q(x) = ck_f\left(\frac{g\beta}{\nu^2}Pr\right)^{1/4} \times \Delta T^{5/3}\left\{\sum_{i=1}^{n}\frac{3}{8k_i}[\Delta T_i^{8/3} - \Delta T_{i-1}^{8/3}]\right\}^{-1/4} \quad (6)$$

where

$$k_i = (\Delta T_i - \Delta T_{i-1})/(s_i - s_{i-1}).$$

If natural convection boundary layer is turbulent it seems that heat transfer is not at least much affected by a streamwise coordinate [5, 6]. In that case the solution can be obtained by usual methods.

3. FORMULATION OF THE MODEL AND SOLUTION PROCEDURE

The energy balance of the fin in the conjugated heat transfer problem of Fig. 1(a) involves heat conduction within the fin in the x- and y-directions and an inflow of heat from convective boundary layer. The temperature and velocity distributions in the boundary layer are three-dimensional. However, the problem is simplified by assuming that convective heat flux can be described using equation (1) for forced and equation (4) for natural convection. This assumption is analogous with the treatment of the condensate film in Patankar and Sparrow [7]. Also if the vertical height of the fin is substantially greater than the transverse length l, the streamwise conduction in the fin is negligible compared with the y conduction. With these assumptions the thin fin energy balance can be written as

$$kt\frac{\partial^2 T}{\partial y^2} - q(x) = 0 \tag{7}$$

where convective heat flux $q(x)$ is governed by equation (1) in the case of forced convection and respectively by equation (4) for natural convection.

Equation (7) cannot be solved without employing the numerical technique. The first step in the search for the solution is to form a finite difference grid of the fin and guess the temperature distribution of the fin (see Fig. 1). If temperature distribution is approximated by straight lines, convective heat flux of the point i, j is easily calculated from equation (2) for forced and from equation (6) for natural convection. When heat flux and temperature are known, the heat transfer coefficient can be obtained. Now attention is paid to the ith vertical row of the fin. Because heat transfer coefficients are known at every point j, a standard finite difference technique can be used to calculate the effect of heat conduction in the y-direction and obtain a new temperature distribution of the ith vertical row. This kind of treatment is repeated in every vertical row of the fin and as a result a new temperature distribution of the fin is obtained. After comparing temperature distribution obtained with that of the initial guess, the procedure is repeated if deviation is too great. The process is repeated until a sufficient degree of accuracy is obtained.

4. RESULTS AND DISCUSSION

The method previously described gives the temperature distribution of the fin and also heat transfer rate, which often is the most interesting quantity when dealing with extended surfaces. In the present problem great generality of heat transfer results can be achieved by comparing them with those of an ideal isothermal fin whose temperature is everywhere uniform and equal to the base temperature T_w.

Forced convection

The temperature distribution of the fin is governed by equation (7). In the case of forced convection it can be changed into dimensionless form, which is free of parameters, by using nondimensional variables defined as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

$$y_* = \frac{y}{l} \tag{9}$$

$$X_* = \frac{1}{C}\frac{kt}{k_f l^2}\frac{x}{Re^m\, Pr^n}. \tag{10}$$

The local heat flux to the fin from the base surface is obtained from the temperature distribution. If only the other side of the fin is considered the governing equation for the local heat flux is

$$\phi'(x) = kt\left(\frac{\partial T}{\partial y}\right)_0. \tag{11}$$

The corresponding local heat flux of an isothermal fin at the station x is written as

$$\phi_i'(x) = C\frac{k_f}{x} Re^m\, Pr^n\, \Delta T_w l. \tag{12}$$

By using dimensionless variables of equations (8)–(10) the local heat flux ratio, if comparing the actual fin with the ideal fin, is evaluated using equations (11) and (12), with the r3sult

$$\frac{\phi'}{\phi_i'} = X_*\left(\frac{\partial \theta}{\partial y_*}\right)_0. \tag{13}$$

The overall rate of heat transfer ϕ from the base over a height from 0 to x is calculated by integrating equation (11)

$$\phi = \int_0^x kt\left(\frac{\partial T}{\partial y}\right)_0 \mathrm{d}x. \tag{14}$$

Comparing the actual total heat transfer with that of the ideal fin of height x, which is

$$\phi_i = C\frac{k_f}{m} Re^m\, Pr^n\, \Delta T_w\, l \tag{15}$$

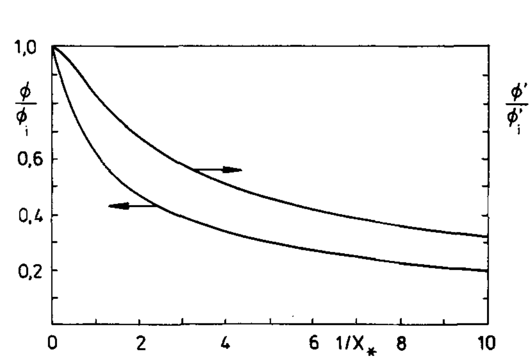

FIG. 2. Local and overall fin heat transfer results. Forced convection with laminar boundary layer.

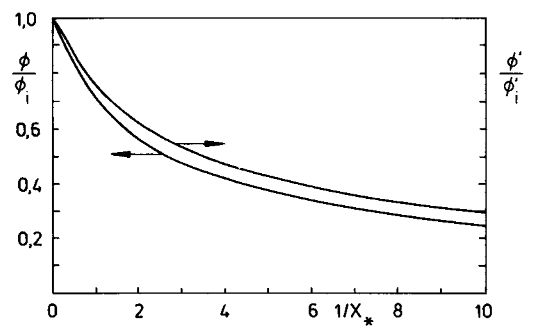

FIG. 3. Local and overall fin heat transfer results. Turbulent boundary layer.

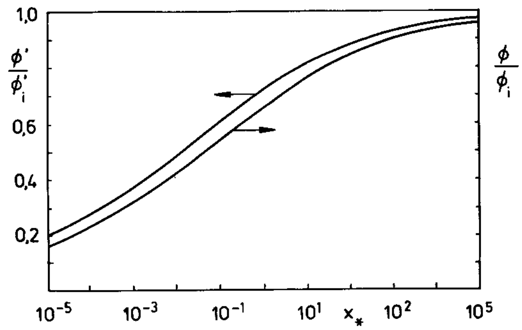

FIG. 4. Heat transfer results for natural convection in a vertical fin.

the ratio ϕ/ϕ_i using dimensionless variables follows as

$$\frac{\phi}{\phi_i} = \frac{m}{1-m} x_*^{\frac{m}{m-1}} \int_0^{x_*} x_*^{\frac{m}{1-m}} \left(\frac{\partial \theta}{\partial y_*}\right)_0 \mathrm{d}x_* \quad (16)$$

Calculated heat flux ratios from equations (13) and (16) are shown in Figs. 2 and 3. Figure 2 presents the results for the laminar boundary layer and Fig. 3 for the turbulent boundary layer.

Natural convection

In the case of natural convection, equation (7) can be transformed into a dimensionless form by introducing dimensionless variables defined by equations (8) and (9) and by using instead of equation (10) a variable

$$x_* = \frac{1}{c^4}\left(\frac{v^2}{g\beta Pr}\right)\frac{1}{\Delta T_w}\left(\frac{kt}{k_f l^2}\right)^4 x. \quad (17)$$

The local heat flux is again obtained from equation (11). When dealing with natural convection, the local heat flux of an isothermal surface at the station x is

$$\phi_i'(x) = ck_f\left(\frac{g\beta}{v^2} Pr\right)^{1/4} \frac{\Delta T_w^{5/4}}{x^{1/4}} l. \quad (18)$$

Comparing the local heat flux with equation (18) the ratio ϕ'/ϕ_i' can be expressed using dimensionless variable of equation (17) as

$$\frac{\phi'}{\phi_i'} = x_*^{1/4}\left(\frac{\partial \theta}{\partial y_*}\right)_0. \quad (19)$$

Equation (14) gives the total heat transfer from the other side of the fin with a height x. The overall heat transfer of an ideal fin is obtained from equation

$$\phi_i = \frac{4}{3} ck_f\left(\frac{g\beta}{v^2} Pr\right)^{1/4} \Delta T^{5/4} x^{3/4} l. \quad (20)$$

Forming the dimensionless ratio ϕ/ϕ_i the final result

$$\frac{\phi}{\phi_i} = \frac{3}{4}\frac{1}{x_*^{3/4}} \int_0^{x_*} \left(\frac{\partial \theta}{\partial y_*}\right)_0 \mathrm{d}x_* \quad (21)$$

is obtained. The ratio of local flux in equation (19) and that of overall heat transfer in equation (21) are plotted in Fig. 4. Also in the case of natural convection experiments were performed using an aluminium fin with a thickness 0.5 mm and which was cooled by air. It was found that experimental results were in agreement with those of Fig. 4.

CONCLUSION

To summarize, the analysis given here predicts heat transfer from a vertical plate fin cooled by natural or forced convection. Very valuable results are obtained by comparing local or total heat transfer with those of an ideal isothermal fin and by using dimensionless variables. It is found that heat transfer of a fin for forced convection with laminar or turbulent boundary layer and for natural convection with laminar boundary layer can be expressed without parameters. Using plotted results of Figs. 2, 3 and 4 actual heat transfer of any fin can easily be evaluated.

REFERENCES

1. D. Q. Kern and A. O. Kraus, *Extended Surface Heat Transfer*. McGraw-Hill, New York (1972).
2. W. M. Rohsenow and J. P. Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973).
3. G. D. Raithby and K. G. T. Hollands, A general method of obtaining approximate solutions to laminar and turbulent free convection problems in *Advances in Heat Transfer*, vol. 11. Academic Press, New York (1975).
4. S. W. Churchill and H. Ozoe, A correlation for laminar free convection from a vertical plate, *J. Heat Transfer* **95**, 540–541 (1973).
5. D. C. Ross and G. C. Vliet, Turbulent natural convection on upward and downward facing inclined constant heat flux surfaces, *J. Heat Transfer* **97**, 549–555 (1975).
6. D. E. Fussey and I. P. Warneford, Free convection from a downward facing inclined flat plate *Int. J. Heat Mass Transfer* **21**, 119–126 (1979).
7. S. V. Patankar and E. M. Sparrow, Condensation on an extended surface, *J. Heat Transfer* **101**, 434–440 (1979).

CONVECTION THERMIQUE NATURELLE OU FORCEE SUR UNE AILETTE PLANE

Résumé—On étudie le transfert thermique d'une plaque plane qui est refroidie par convection naturelle ou forcée. On utilise des expressions approchées pour relier le flux convectif et la température dans le cas de la couche limite laminaire ou turbulente en écoulement forcé et dans le cas d'une convection naturelle dans une ailette verticale. On présente une procédure simple de résolution pour le transfert thermique couplé de conduction dans l'ailette et de convection sur elle. Des résultats sont obtenus dans la comparaison d'une ailette réelle et d'une ailette isotherme idéale. On trouve qu'une simple courbe représente correctement les résultats sans paramètres.

WÄRMEÜBERGANG BEI FREIER UND ERZWUNGENER KONVEKTION AN EINER EBENEN RIPPE

Zusammenfassung — Der Wärmeübergang an einer durch erzwungene oder freie Konvektion gekühlten ebenen Rippe wird untersucht. Zur Verknüpfung des konvektiven Wärmestroms mit der Temperatur bei laminarer und turbulenter Grenzschicht in erzwungener Strömung und bei freier Konvektion an einer senkrechten Rippe werden Näherungsbeziehungen verwendet.

Zur Behandlung der verknüpften Wärmetransportvorgänge durch Wärmeleitung in einer Rippe und durch konvektiven Wärmeübergang wird ein einfaches Lösungsverfahren vorgestellt. Der Vergleich des Wärmetransports in einer tatsächlichen Rippe mit dem in einer ideal isothermen ergibt deutliche Unterschiede. Es zeigt sich, daß zur Darstellung des Wärmetransports eine Kurve ausreicht und daß keine Parameter erforderlich sind.

ТЕПЛООБМЕН ПРИ ЕСТЕСТВЕННОЙ И ВЫНУЖДЕННОЙ КОНВЕКЦИИ НА ПЛОСКОМ РЕБРЕ

Аннотация — Анализируется теплообмен на плоском ребре, которое охлаждается посредством вынужденной или естественной конвекции. Установлены приближённые соотношения между конвективным тепловым потоком и температурой для ламинарного и турбулентного пограничного слоя при вынужденном течении и для естественной конвекции на вертикальном ребре. Представлена простая методика расчёта сопряжённого теплообмена, учитывающая теплопроводность ребра и конвекцию на нём. Получены значимые результаты по теплообмену для случая, когда теплообмен реального ребра сравнивается с идеальным изотермическим ребром. Обнаружено, что одной кривой достаточно для представления результатов по теплообмену без параметров.